Summary: Scaling Laws

The Scaling Laws inquiry activity was a part of Research Saturday at UC Santa Cruz. 16 undergraduates (volunteer participants from PHYS 6A and PHYS 7A) participated. The content learning outcome for this activity was to have students understand how physical properties scale with size -- specifically, we wanted students to be able to deduce how physical properties of objects change with size using geometrical arguments (such as ratios between volume, surface area, and/or length). The core practice we focused on was properly designing and controlling experiments.

The activity took approximately 5 hours to complete. Students were given a scenario: a very petty (and very wealthy) real estate developer sees a competitor build a very large building, and wants to outdo her. He takes her blueprint, *multiplies every linear dimension by two*, and gives the blueprint to his engineers. However, the engineers notice some problems. Students are split into three groups which rotate through the following stations:

- Lighting: All bulbs in the building will be twice the distance to the ground. By using a set of three identical light bulbs and a barrier in a dark room, we demonstrate that:
    - A light that is 50cm from the ground illuminates the surface less than a light 25cm from the ground
    - *Two lights* 50cm from the ground illuminate the surface less than a single light 25cm from the ground.
- Electrical: The size of the wires in the building will be doubled as well. By using two simple light bulb circuits which are identical except for a short length of nichrome wire, we compare the effect of a “large size” wire to a “short size” wire. We demonstrate that:
    - When the wire is larger, the bulb shines more brightly

  After a brief discussion of the phenomenon of electrical resistance, we also measure directly the resistance of the two segments of nichrome wire, demonstrating:
    - The larger wire has a lower resistance
    - This coincides with the brighter shining light bulb
- Structural: In the original building, there is a ballroom with a chandelier held up by a pair of wires. By using fishing line, calipers, and garbage bags with dirt (“chandeliers”), we demonstrate that:
    - A 10cm chandelier is supported easily by a 0.01in diameter fishing line
    - A 15cm chandelier is supported tenuously by a 0.015in diameter fishing line
    - An 18cm chandelier breaks 0.018in diameter fishing line

While at the stations, participants are encouraged to write questions they would like to investigate about the phenomena they observe on a communal sheet of poster board. Once all three groups have visited all the stations, the poster boards are hung in the front of the room, and participants identify their first choice and second choice for the phenomena they wish to investigate. Facilitators then jigsaw students into “universities” and “conference groups”. Each university has three students, one from each conference group. This structure gives each student a unique role, and fosters the development of STEM identity.

After a brief activity with the university groups, students split into conference groups and investigate:

- Lighting: Students were put in a dark room with different types of lights, tracks (with length markings), and photometers with voltmeters to measure light intensity. Possible investigation pathways:
    - Find light intensity as a function of distance from light source
    - Find light intensity as a function of number of bulbs
    - Find light intensity as function of scale factor (e.g. double bulbs and double distance)
- Electrical: Students were provided with nichrome wire of varying gauges, calipers, wirecutters, rulers, and ohmmeters. Possible investigation pathways:
    - Find resistance of wire as a function of wire length
    - Find resistance of wire as a function of gauge
    - Find resistance of wire as a function of scale factor
- Structural: Students are provided with six different gauges of fishing line, calipers, a scale, and hanging weights. Possible investigation pathways:
    - Test breaking point of wire as a function of wire gauge
    - Test breaking point of wire as a function of length
    - Test weight of objects as a function of their size
        - This was impractical here, but might be doable in the future -- it would require heavy objects that are mathematically similar. Bags of dirt are difficult to measure.
    - Find breaking point as a function of scale factor for different starting masses (hard)

For all investigations, facilitators checked in frequently to make sure experiment questions were well-defined, and that experiments were well-controlled. Once students gathered some relevant data, facilitators begin to ask students to explain the data, gradually moving to a scaling explanation of the phenomenon based purely in geometry.

At the end of the investigation, students are put into their university groups for the culminating assessment. Each university is instructed to submit a report that addresses the following prompts:

Prompt 1: Explain how the electrical, structural, and lighting systems are affected by doubling or halving the size of a building. Justify your answers using experimental evidence (presented as graphs).

Prompt 2: Let a be a measure of how much the building is scaled -- for example, if the building is doubled in size, a = 2, and if the building size is halved, a = ½. What type of function (e.g. f(a) ~ a^2, f(a) ~ a^{-3}, f(a) ~ e^a) fits your data the best?

Prompt 3: Explain using geometric arguments (that is, without taking your data into consideration) why the function type in the previous prompt must be the correct function type. Most good explanations are expressed in terms of simpler, intermediate quantities that lead to a single scaling law.

This report is the artifact by which we assessed the participants. A second culminating assessment was planned (an individual assignment in which learners try to derive a new scaling relationship), but we ran out of time. The average scores for the culminating assessment were:

Structural: Practice 6.5/9*, Content 7.2/12
Electrical: Practice 5.2/9, Content 8.4/12
Lighting: Practice 5.4/9, Content 7/12
Overall: Practice 5.64/9, Content 7.53/12

*One report did not present evidence for practice dimensions, and was omitted from the average.